

# To Hash or Not To Hash Again?

(In)differentiability Results for  $H^2$  and HMAC

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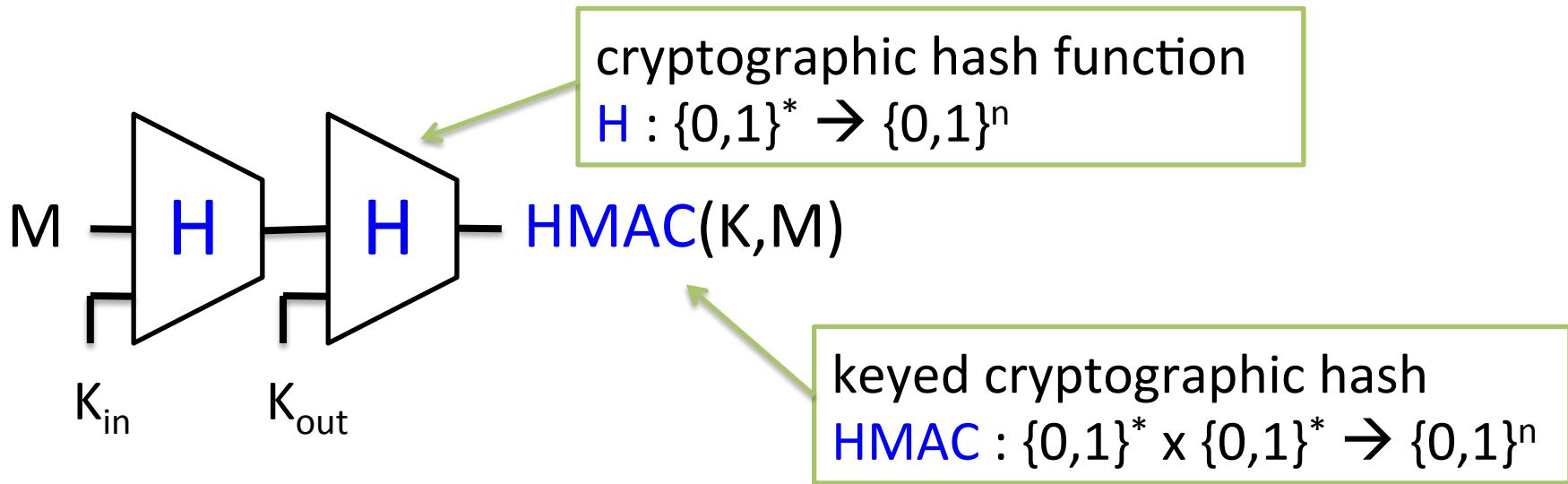
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(Tsinghua University)

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(MIT)



## HMAC construction

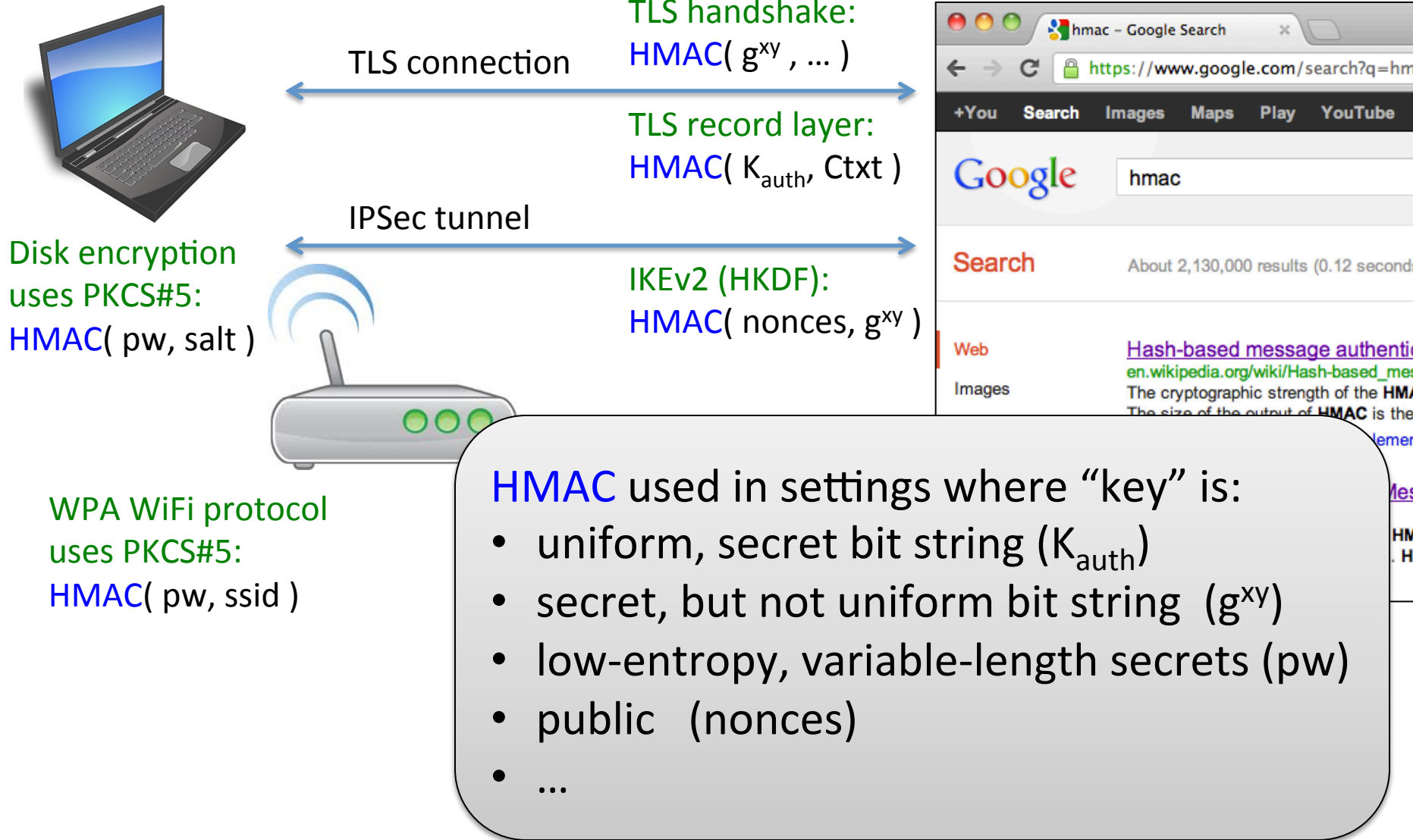
[Bellare, Canetti,  
 Krawczyk 1996]

$K_{in}$ ,  $K_{out}$  derived from  $K$  (details to come)

$$HMAC(K, M) = H( K_{out} \parallel H(K_{in} \parallel M) )$$

Designed for message authentication with secret keys ...  
 ... but now used in a variety of ways

# Usage of HMAC



# Security of HMAC?

When  $K$  is secret, uniform, fixed-length bit string?

Standard-model proofs of PRF/unforgeability security

[Bellare, Canetti, Krawczyk 1996] [Bellare 2006]

When input includes high-entropy, secret bit string?

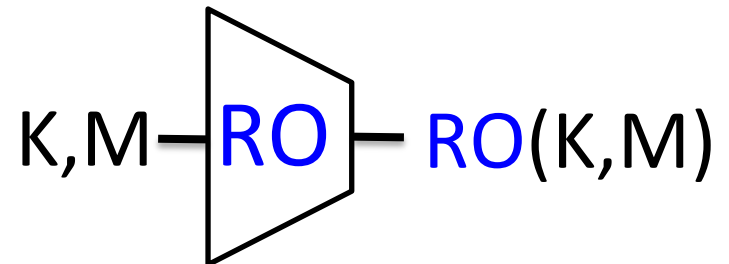
Standard-model proofs that HMAC is a randomness extractor

[DGKHR 2004] [FPZ08] [K10]

# Security of HMAC?

## *Elsewhere?*

Proofs model **HMAC** as a (keyed) random oracle (**RO**) [BR93]



Each pair  $K, M$  mapped to uniformly chosen output  $\text{RO}(K, M)$

Examples:

HKDF/IKEv2 [K10]

$\text{HMAC}(\text{salt}, S)$

Public value

PKCS#5 [BRT12]

$\text{HMAC}(\text{pw}, \text{salt})$

Non-uniform, varying length secret. Often low entropy.

Hedged crypto [RY10]

$\text{HMAC}(R, M)$

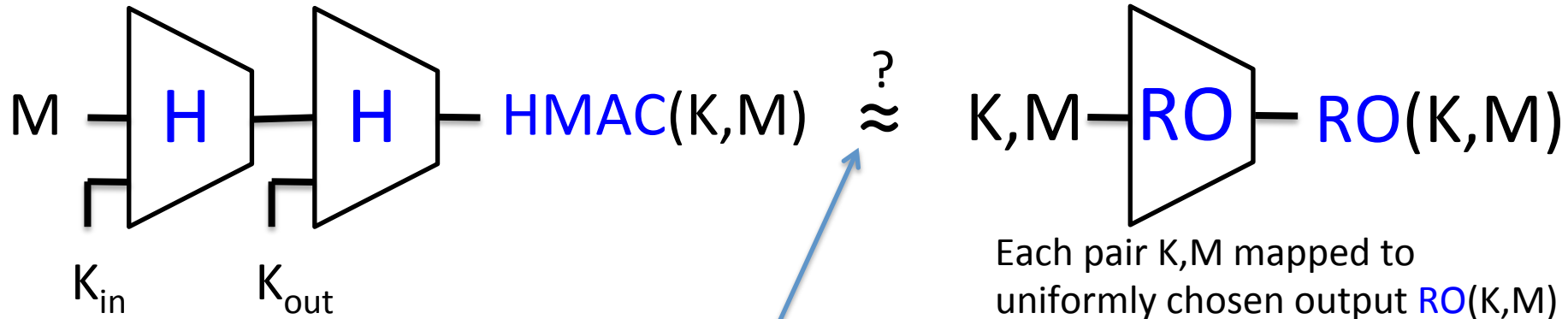
Adversarially controlled

Practitioners expect that **HMAC** behaves “randomly” for any  $K, M$

# HMAC as a RO

Let's (generously) assume  $H$  is perfectly secure (itself a random oracle)

Does  $HMAC$  "behave like" a  $RO$ ?

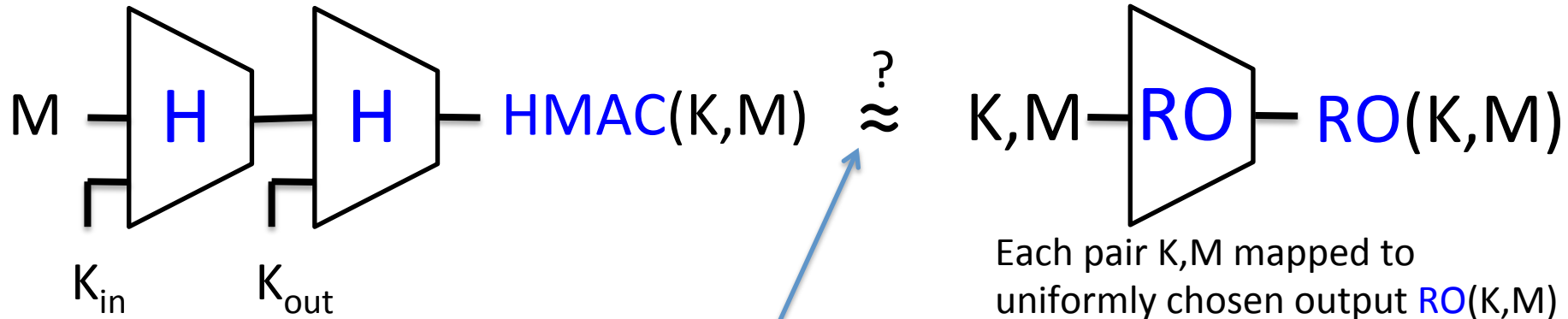


Formalize "behave like"  $RO$  via indistinguishability [MRH04]

# HMAC and $H^2$ as ROs

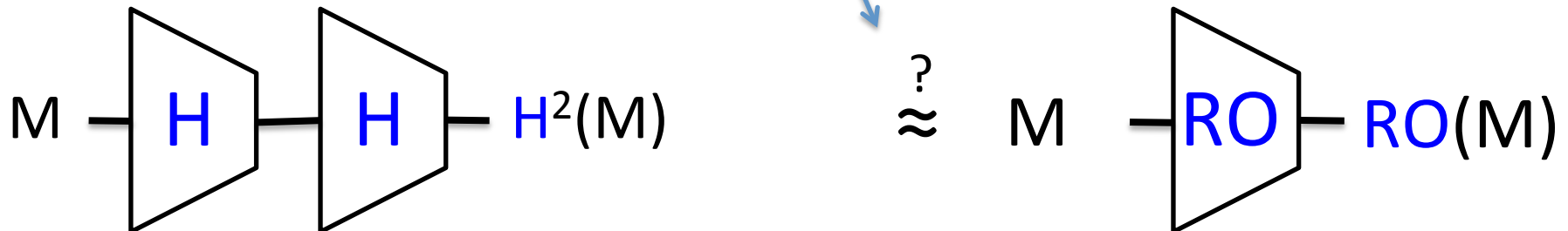
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Formalize "behave like" **RO** via indistinguishability [MRH04]

Does  $H^2$  "behave like" a **RO**?



"Practical cryptography" [Ferguson, Schneier 2003]

Each pair  $M$  mapped to uniformly chosen output  $RO(M)$

# One expects the answers to be “yes”

Strong positive intuition:

- Both designed to prevent length-extension attacks
- Compose **RO** with itself, result should behave like a **RO**

Confusingly, they refer to this as the “HMAC Construction”. It’s not **HMAC**

[CDMP05] prove  $H^2(0^d || M)$  indifferentiable from a **RO**, when **H** is Merkle-Damgard-style hash function (e.g., SHA-256)

[K10] suggests that the [CDMP05] proof extends to the real **HMAC**



# Summary of our results

Uncover that **HMAC** has *weak key pairs*

(1) **Colliding key pairs**       $K \neq K'$  s.t.  $\text{HMAC}(K, M) = \text{HMAC}(K', M)$

(2) **Ambiguous key pairs**      no inner/outer **H** domain separation

$K \neq K'$  s.t.  $K_{\text{in}} = K'_{\text{out}}$        $K'_{\text{in}} = K_{\text{out}}$

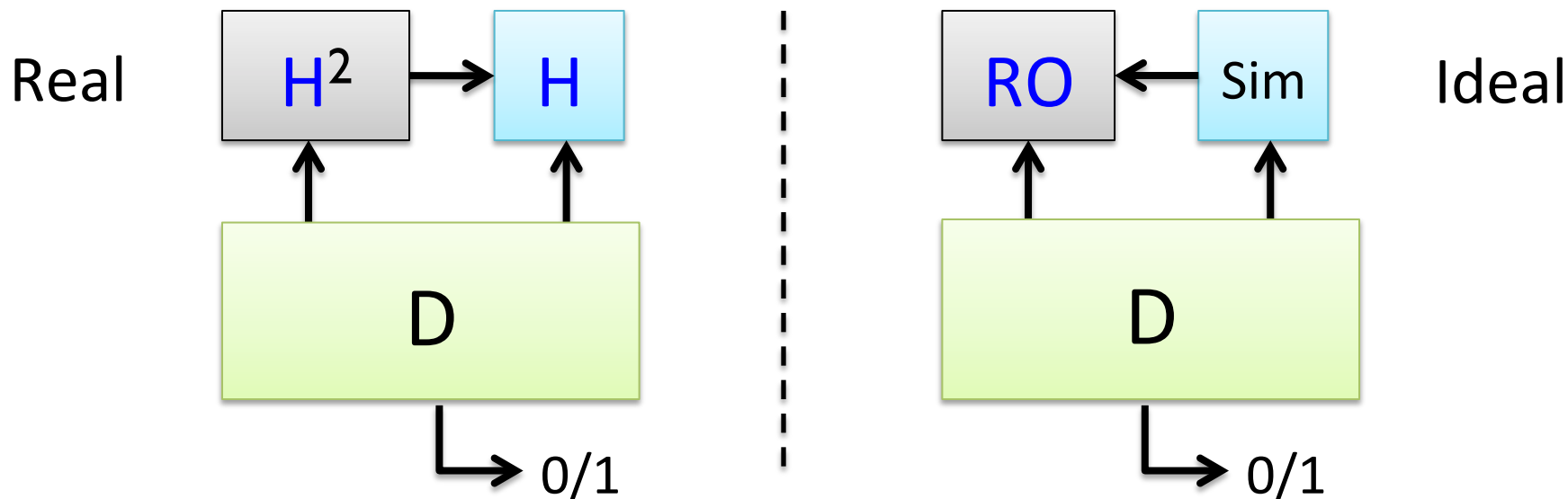
$H^2$  and **HMAC** w/ **ambiguous key pairs** have similar issues

- At best weak concrete security from indifferentiability
- Upper bound proof that  $H^2$  is “weakly” indifferentiable
- Example (vulnerable) setting: **mutual proofs of work**

Avoid weak key pairs in **HMAC**, get **strong indifferentiability**

[CDMP05]: formalize “behave like a RO” via  
indifferentiability framework of [MRH04]

### *Indifferentiability from a RO*



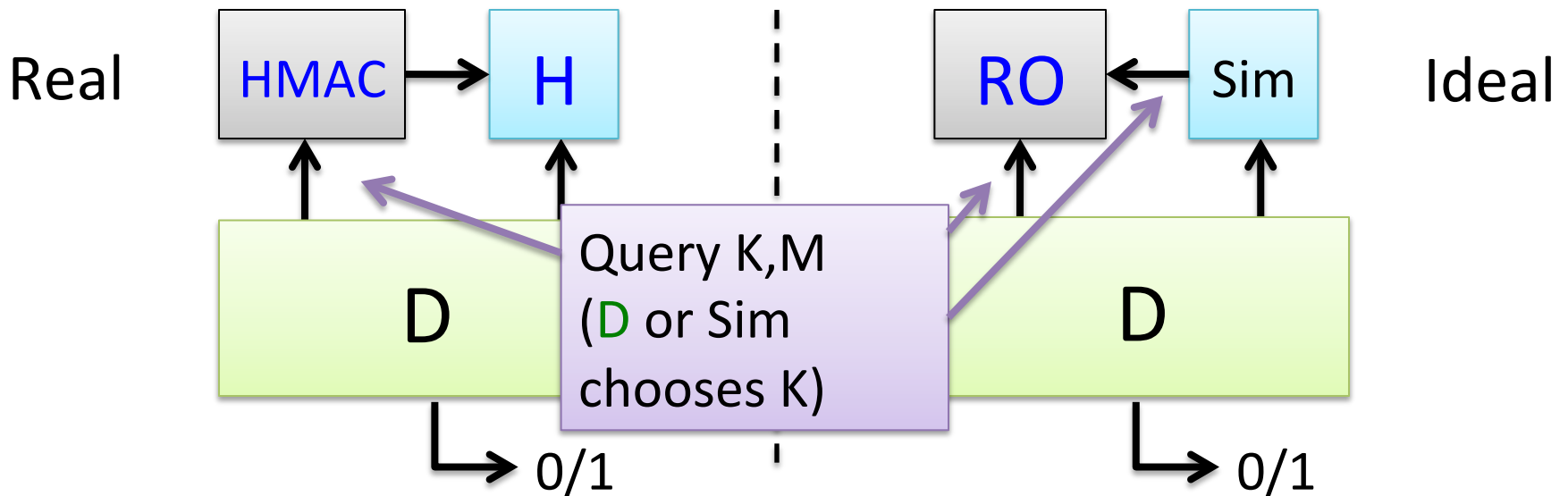
**H**, **RO** both random functions

**Indifferentiable** if exists efficient **Sim** s.t. for all efficient **D**

$$\text{Adv}^{\text{indiff}}(\mathbf{H}^2, \mathbf{D}, \text{Sim}) = \Pr[\text{Real}(\mathbf{D}) \Rightarrow 1] - \Pr[\text{Ideal}(\mathbf{D}) \Rightarrow 1] < \varepsilon$$

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*Indifferentiability from a RO*



$H$ ,  $RO$  both random functions

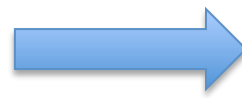
**Indifferentiable** if exists efficient  $Sim$  s.t. for all efficient  $D$

$$Adv^{indiff}(H^2, D, Sim) = \Pr[ \text{Real}(D) \Rightarrow 1 ] - \Pr[ \text{Ideal}(D) \Rightarrow 1 ] < \epsilon$$

$$Adv^{indiff}(HMAC, D, Sim) = \Pr[ \text{Real}(D) \Rightarrow 1 ] - \Pr[ \text{Ideal}(D) \Rightarrow 1 ] < \epsilon$$

[CDMP05]: formalize “behave like a RO” via  
indifferentiability framework of [MRH04]

(Conjectured) HMAC  
indifferentiability  
+  
[BRT12],[K10],[RY10] proofs  
of security for applications  
using RO



[MRH04]  
composition  
theorem

Applications proven  
secure using HMAC  
(H still ideal)

Rules out attacks that  
abuse structure of HMAC

Limitations:

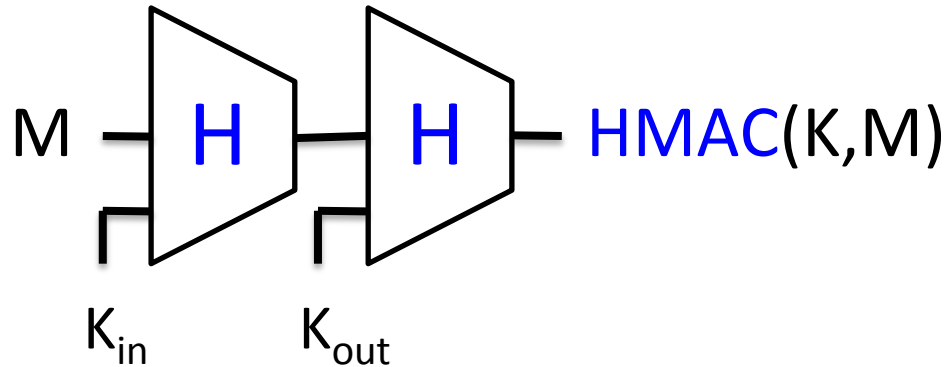
- [RSS11]: composition only applies to “single-stage” games
- Concrete security of indifferentiability important!

**Normal indifferentiability:** Sim makes  $O(q)$  queries

**Weak indifferentiability:** Sim makes  $>O(q)$  queries

# A closer look at HMAC

Let  $H : \{0,1\}^* \rightarrow \{0,1\}^n$  be parameterized by a block length  $d \leq n$



HMAC(K,M):

If  $|K| > d$  then  $K' \leftarrow H(K)$

else  $K' \leftarrow K$

$K'' \leftarrow K' \parallel 0^{d-|K'|}$

$K_{in} \leftarrow K'' \oplus \text{ipad}$

$K_{out} \leftarrow K'' \oplus \text{opad}$

Return  $H(K_{out} \parallel H(K_{in} \parallel M))$

ipad  $\neq$  opad fixed strings

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# A closer look at HMAC

Let  $H : \{0,1\}^* \rightarrow \{0,1\}^n$  be parameterized by a block length  $d \leq n$

## 1) Colliding key pairs

Any  $K_1 \neq K_2$  such that

$$\text{HMAC}(K_1, M) = \text{HMAC}(K_2, M)$$

Example:  $K_2 = K_1 \parallel 0 \quad |K_1| < d$

Simple distinguisher breaking  
indifferentiability with 2 queries

Possible security issue anywhere  
variable-length keys used

No colliding keys if use  
fixed-length keys (and  $H$  is CR)

HMAC(K,M):

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D

Query left oracle on  $(K, M)$  to get  $Y$

Query left oracle on  $(K \parallel 0, M)$  to get  $Y'$

If  $Y = Y'$  then ret 1

ret 0

# A closer look at HMAC

Let  $H : \{0,1\}^* \rightarrow \{0,1\}^n$  be parameterized by a block length  $d \leq n$

## 1) Colliding key pairs

Any  $K1 \neq K2$  such that

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else  $K' \leftarrow K$

$K'' \leftarrow K' \parallel 0^{d-|K'|}$

$K_{in} \leftarrow K'' \oplus \text{ipad}$

$K_{out} \leftarrow K'' \oplus \text{opad}$

Return  $H(K_{out} \parallel H(K_{in} \parallel M))$

ipad  $\neq$  opad fixed strings

## 2) Ambiguous key pairs

Any  $K1 \neq K2$  such that

$$K1_{in} = K2_{out} \quad K2_{in} = K1_{out}$$

Example:  $K2 = K1 \oplus \text{ipad} \oplus \text{opad} \quad |K1| = |K2| = d$

For all X:

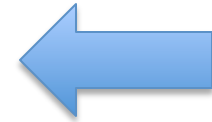
$$H(K1_{in} \parallel X) = H(K2_{out} \parallel X)$$
$$H(K2_{in} \parallel X) = H(K1_{out} \parallel X)$$

No inner/outer H domain separation.



# Summary of HMAC indifferenciability

Key space includes	Indifferentiable?
Colliding key pairs	No
Ambiguous key pairs (no colliding key pairs)	At most weak
Keys K of fixed length $ K  < d-1$	Yes



→ Avoids both kinds of weak key pairs:

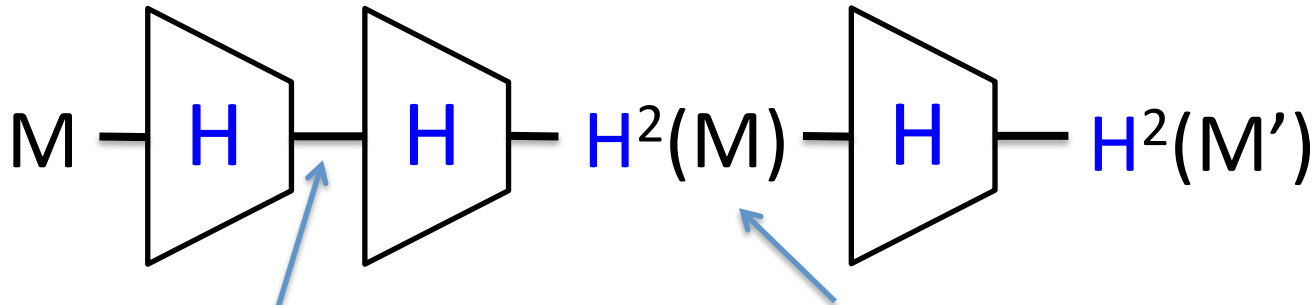
## Indifferentiability upper bound (simplified)

For HMAC with restricted keys, exists Sim such that for any D

$$\text{Adv}^{\text{indiff}}(\text{HMAC}, D, \text{Sim}) < O(q^2 / 2^n)$$

Sim makes  $O(q)$  queries

# Lack of inner/outer domain separation



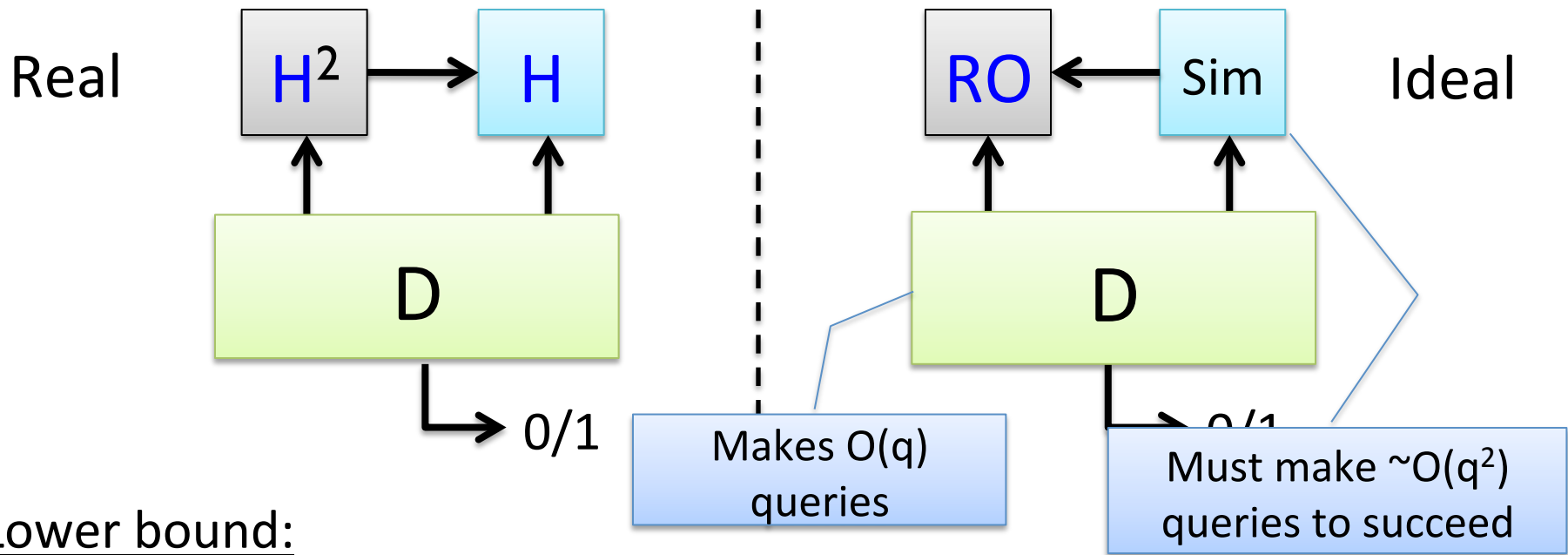
$$M' = H(M)$$

Output of  $H^2$  "leaks" an internal value for another hash computation

## Worrisome:

- Prior indifferentiable constructions avoid leaking intermediate values
- Extension attacks abused such leaks

# Indifferentiability lower and upper bounds (simplified)



## Lower bound:

We give  $D$  making  $q_L$  left queries and  $q_R$  right queries such that for any  $Sim$  making  $q_s$  queries:

$$\text{Adv}^{\text{indiff}}(H^2, D, Sim) > 1 - q_s / (q_L q_R)$$

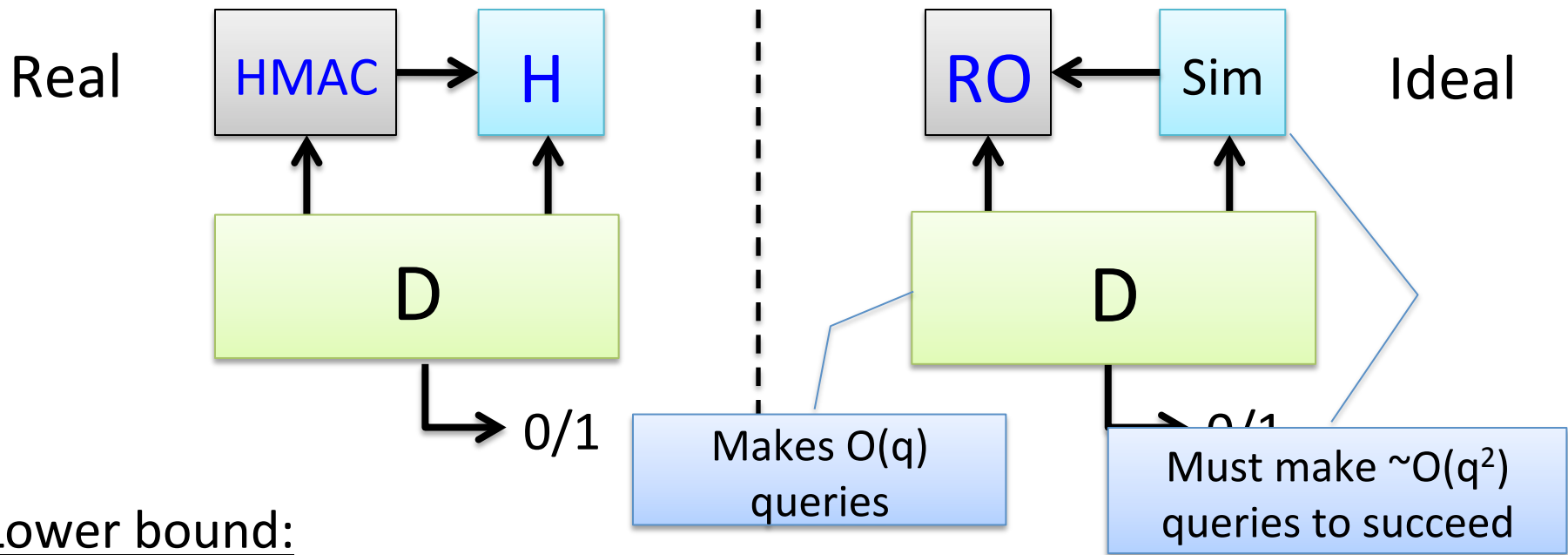
## Upper bound:

We give  $Sim$  such that for any  $D$  making  $q$  queries

$$\text{Adv}^{\text{indiff}}(H^2, D, Sim) < q^2 / 2^n$$

Sim makes  $O(q^2)$  queries

# Indifferentiability lower and upper bounds (simplified)



## Lower bound:

We give  $D$ , making  $q_L$  left queries and  $q_R$  right queries including **ambiguous key pairs**, such that for any  $Sim$  making  $q_S$  queries:

$$\text{Adv}^{\text{indiff}}(\text{HMAC}, D, \text{Sim}) > 1 - q_S / (q_L q_R)$$

## Upper bound: ???

Indifferentiability for **ambiguous key pairs** (but no **colliding keys**) probably holds with  $Sim$  making  $O(q^2)$  queries. We don't have a proof.

# Does $q^2$ versus $q$ really matter?

**In paper we provide an example:  
mutual proofs of work protocol**

**vulnerable** using  $H^2$   
or **HMAC** with  
ambiguous key pairs

**secure** using **RO**

# Vulnerabilities in practice?

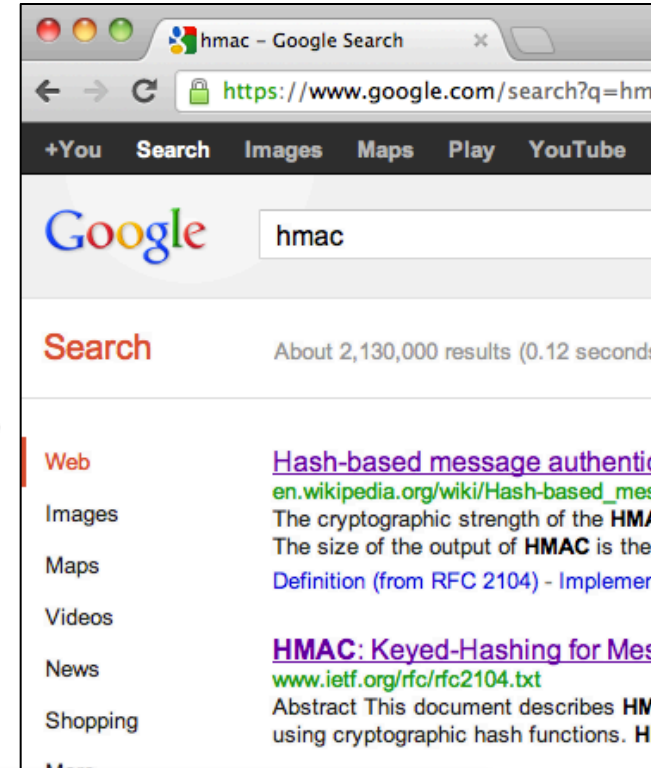
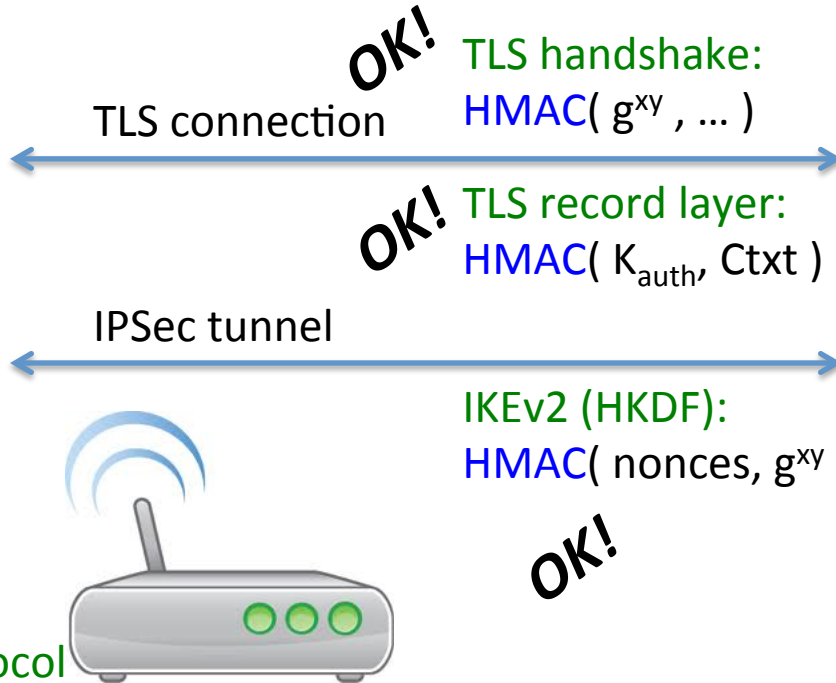


Disk encryption  
uses PKCS#5:  
 $\text{HMAC}(\text{pw}, \text{salt})$

???

WPA WiFi protocol  
uses PKCS#5:  
 $\text{HMAC}(\text{pw}, \text{ssid})$

???



Most applications already  
avoid weak key pairs

# Discussion

Results extend to when  $H$  is an iterative hash function (e.g., SHA-1, SHA-256, ...)

In theory, we can fix  $H^2$  and  $HMAC$ , but deployment would be a ... hurdle

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