

# How to Compute under $AC^0$ Leakage without Secure Hardware

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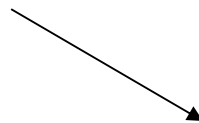
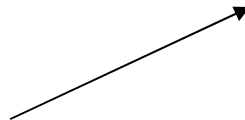
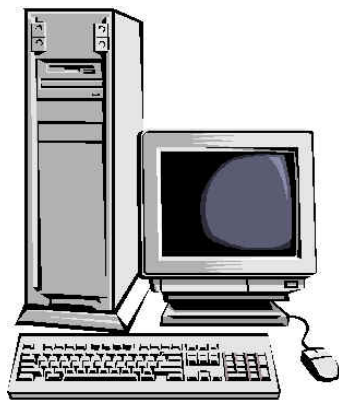
# Protecting Sensitive Computations from Leakage/Side-Channel Attacks

## Sensitive computations:

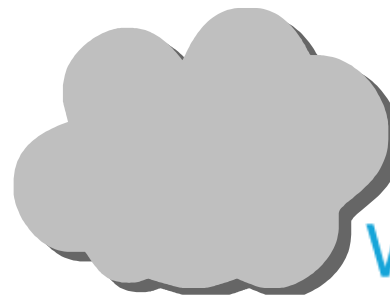
- Cryptographic Algorithms
  - Secret Key
- Proprietary Search Algorithm,  
Private Medical Data Base Processing...
  - Secret Program, Data

# ... are Performed Remotely

## Mobile Devices



## Remote Computing



Windows Azure Platform

# Computation Internals Might Leak



EM Radiation

[Quisquater 01]



Power

Consumption

[Kocher et al. 98]



Cache [Kocher 96]



Timing [Kocher 96]

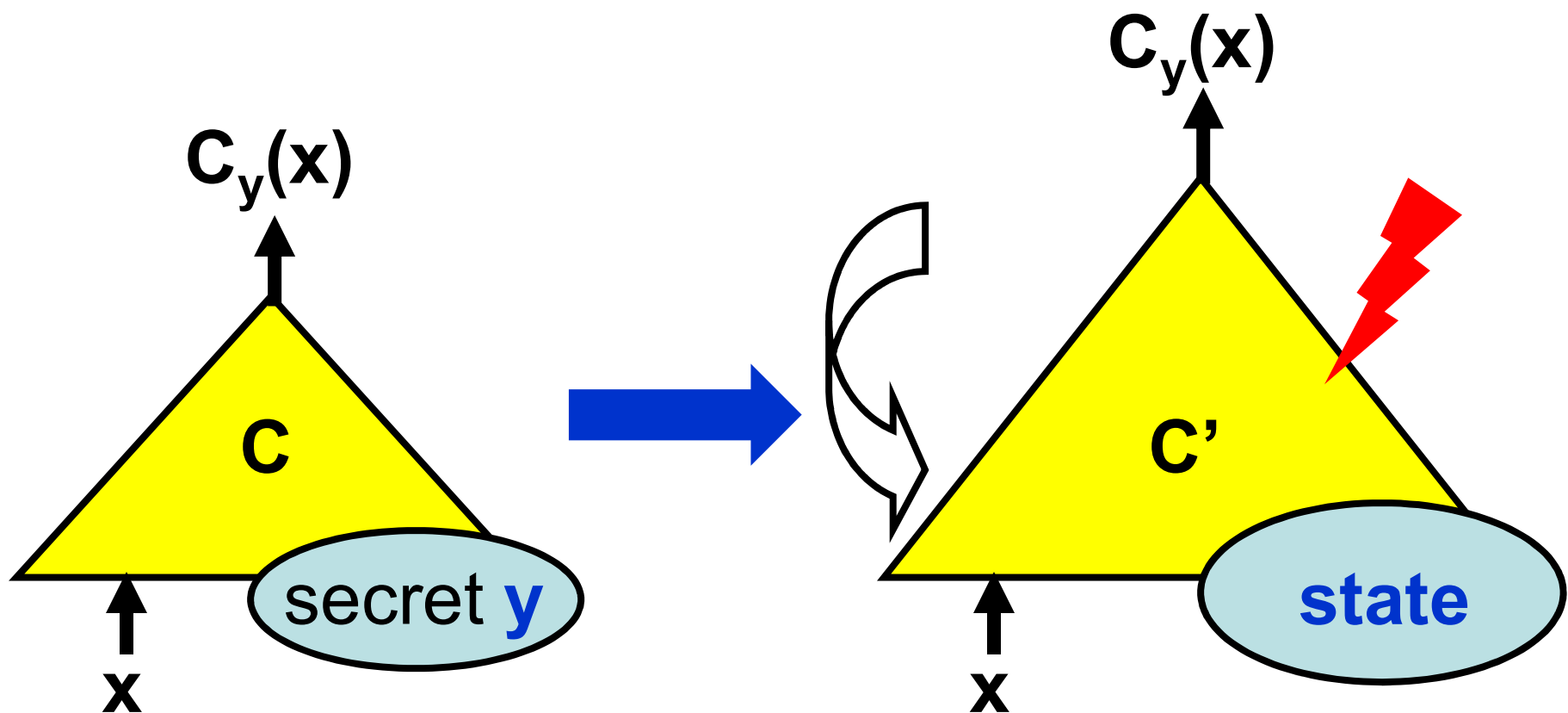
# Two Approaches to Fighting Leakage Attacks

- Consider leakage at **design time**  
[AGV09,...]  
build systems secure against leakage attacks

## HOLY GRAIL

- “**Leakage resilience compiler**”  
[GO96, ISW03,...]  
transform **any** algorithm so that,  
even under leakage,  
no more than black-box behavior is exposed

# Our Goal: Leakage-Resilience Compiler



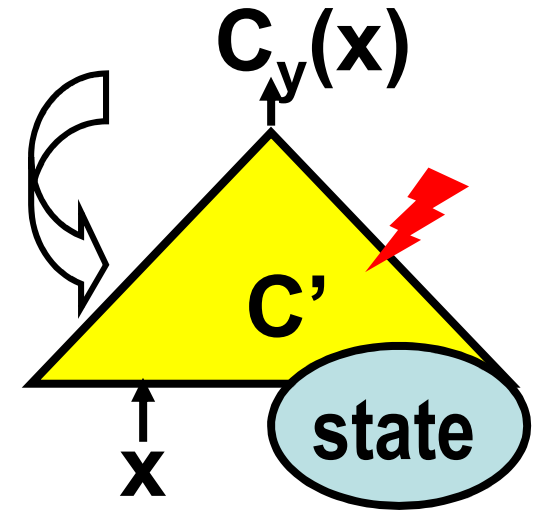
Even given leakage,  
execution “looks like”  
**black-box access to  $C_y(x)$**

# Offline/Online Leakage Model

**Offline** (only once): no leakage

Process **C** and **y**

$s_1 \leftarrow \text{Init}(\mathbf{C}, \mathbf{y}, r_0)$



**Online**, in each execution  $t \leftarrow 1, 2, 3, \dots$

Adv chooses input  $x_t$

$\text{output}_t \leftarrow C'(x_t, s_t, r_t), s_{t+1} \leftarrow \text{Update}(s_t, r_t)$

Adv observes:  $\text{output}_t + \text{Leakage}_t(x_t, s_t, r_t)$

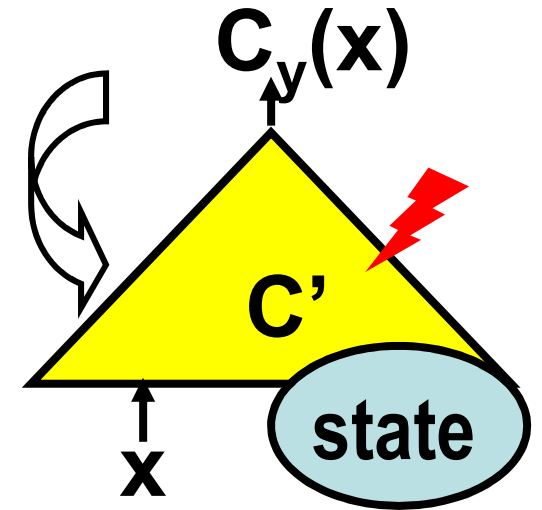
**Leakage<sub>t</sub>**: leakage function chosen from class of permissible functions

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**Leakage<sub>t</sub>**: In this work -  $AC^0$  function  
with bounded output length



# What is $AC^0$ ?

A function  $L$  is in  $AC^0$  if it can be computed by a **poly-size  $O(1)$  depth boolean circuit** with unbounded fan-in AND, OR (and NOT) gates

Some known lower bounds on  $AC^0$

- can't compute parity of  $n$  bits [H86]
- can't compute inner product of  $n$ -bit vectors
- can't “**compress**” parity or inner product [HN10,DI06]

# New Result: Compiler for $AC^0$ Leakage

Can transform any poly time  $C_y$  into  $C'$

On security parameter  $\kappa$ :

1. **Leakage<sub>t</sub>** is  $AC^0$ , output bound =  $\lambda(\kappa)$  bits
2.  $|C'| = O(\kappa^3 \cdot |C|)$
3. Assuming the  $\lambda$ -IPPP assumption, exists simulator **SIM**, s.t.

$$\mathbf{VIEW}_{\text{Leakage}}(C') \approx \mathbf{SIM}^{C_y}$$

# $\lambda$ -IPPP Assumption

Known limits on power of  $AC^0$  circuits: [H86,DI06]  
given  $\mathbf{x}, \mathbf{y} \in \{0,1\}^k$ , can't compute or compress  
 $\langle \mathbf{x}, \mathbf{y} \rangle$  using an  $AC^0$  circuit

$\lambda$ -Inner Product w. Pre-Processing (IPPP) assump

1. poly time to pre-process  $\mathbf{x} \Rightarrow \mathbf{f}(\mathbf{x})$
2. poly time to pre-process  $\mathbf{y} \Rightarrow \mathbf{g}(\mathbf{y})$
3. given  $\mathbf{f}(\mathbf{x}), \mathbf{g}(\mathbf{y})$ , can't compute or compress  
 $\langle \mathbf{x}, \mathbf{y} \rangle$  to  $\lambda(n)$  bits using an  $AC^0$  circuit

Long standing open problem in complexity theory

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# Prior Work on General Compilers

## “Wire-probe” (either/or) leakage functions

[ISW 03],[A10] no hardware, unconditional

## “Local” (OC) leakage functions [MR04]

[GR10],[JV10] secure hardware + crypto

[DF12] secure hardware, unconditional

[GR12] no hardware, unconditional

## AC<sup>0</sup> leakage functions

[FRRTV10] secure hardware, unconditional

# Compiler: High-Level View

(a la [ISW03],[FRRTV10])

- **Init** – “encrypt” bits of **y**  
**Enc(b)**  $\Rightarrow$  “bundle of bits” - random vector, parity **b**  
(**AC**<sup>0</sup> leakage cannot determine parity)
- **Single execution**  
Homomorphically compute on “bundles”  
(computation not in **AC**<sup>0</sup>, but resists **AC**<sup>0</sup> leakage,  
secure hardware used for “blinding”)
- **Multiple executions**  
leakage on bundles encrypting **y** might accumulate  
(secure hardware used to “refresh” bundles)

# [FRRTV10] Secure Hardware

## Functionality:

generates a random bundle with parity **0**

**assume: no leakage on generation procedure**

## Security:

simulator can create view where the bundle parity is **1**, **AC<sup>0</sup>** leakage can't tell the difference

## Uses in the construction:

- “blinding” homomorphic computations
- refreshing **y** bundles between executions

# New Tool: “Bundle Bank” (a la [GR12])

“Realize secure hardware”, even though  
**leakage operates also on generation procedure**

## Functionality:

generate bundles  $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_T$ , s.t. parity  $\mathbf{v}_i = \mathbf{0}$

## Security:

Simulator on input  $(\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_T)$

generate bundles  $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_T$ , s.t. parity  $\mathbf{v}_i = \mathbf{b}_i$

$\mathbf{AC}^0$  leakage on **REAL** and **SIM** is statistically close



# Generating One New Bundle

**Init** (no leakage):

choose  $m$  bundles  $\mathbf{c}_1 \dots \mathbf{c}_m$  with parity  $\mathbf{0}$

**Generating  $\mathbf{c}_{\text{new}}$**  (under leakage):

take random linear combination  $\mathbf{r}$

The diagram shows a matrix  $\mathbf{C}$  on the left, represented as a light blue rectangle with vertical lines separating columns. The first three columns are visible, followed by an ellipsis, and then the last column. Below it is the label  $\mathbf{C} = [\mathbf{c}_1, \dots, \mathbf{c}_m]$ . To the right of  $\mathbf{C}$  is a large 'X' symbol. Next is a vertical light blue bar representing a vector  $\mathbf{r}$ , with the label  $\mathbf{r} \in \{0, 1\}^m$  below it. To the right of  $\mathbf{r}$  is an equals sign. Finally, on the far right, is another vertical light blue bar representing the resulting vector  $\mathbf{c}_{\text{new}}$ , with the label  $\mathbf{c}_{\text{new}}$  below it.

# Simulated Generation

**Init** (no leakage):

~~choose  $m$  bundles  $c_1 \dots c_m$  with parity  $0$~~   
parities are **random**:  $x \in \{0,1\}^m$

**Generating  $c_{\text{new}}$**  (under leakage):

~~take random linear combination  $r$~~   
take **biased** linear combination  $r$  s.t.  $\langle x, r \rangle = b$   
( $\Rightarrow c_{\text{new}}$  parity equals  $b$ )

**Secure?**

**AC<sup>0</sup>** leakage can't tell if  $c_i$ 's have parity  $0$  or  $1$ ,  
and can't tell if  $r$  used in generation is biased

# Bundle Bank Security

Consider  $AC^0$  leakage on **REAL** and **SIM** generating a sequence of **0**-bundles

**Want:**  $AC^0$  security reduction from parity to distinguishing **REAL** and **SIM**

**Obstacle:** generation procedure not in  $AC^0$  (nor are many other computations in construction)

**Our main technical contribution:**

$AC^0$  security reduction from IPPP to distinguishing leakage on **REAL** and **SIM**

Why IPPP? Use pre-processing to set up views

# THANK YOU!

- Compiler transforms **any** computation into one that resists **AC<sup>0</sup>** leakage (under IPPP assumption)
- Strong black-box security
- Secure hardware is not needed

## Questions

- IPPP assumption
- Constant leakage rate
- Connections to obfuscation
- Other leakage classes