Message Authentication, Revisited

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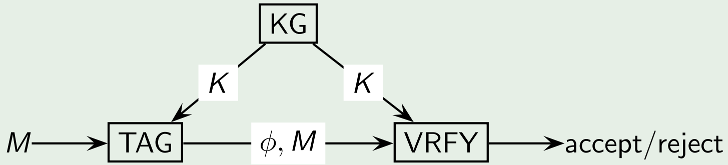
EUROCRYPT 2012, April 16th, Cambridge UK

Message Authentication Codes

MAC = {KG, TAG, VRFY}



- MACs are fundamental cryptographic primitives.
- Historically constructed from PRFs (with large range)

$$\mathsf{TAG}(K, M) \sim \mathsf{PRF}(K, M) \ , \ \ \mathsf{VRFY}(K, M, \phi) \ \sim \ \mathsf{PRF}(K, M) \stackrel{?}{=} \phi$$

- Domain extension: CBC, HMAC, Hash-then-Encrypt...
- Heuristic: AES, SHA,. . .
- Algebraic: Naor-Reingold PRF, LWE-PRF [BPR'12],... less efficient, but provably secure & ZK-friendly (e.g. for e-cash.)

Algebraic MAC/PRF

- The Naor-Reingold PRF (based on DDH in $\mathbb{G}$)

$$F_{NR}(\underbrace{[h, x_1, \ldots, x_m]}_{\text{key } \in \mathbb{G}\times\mathbb{Z}_p^m}, \underbrace{[b_1, \ldots, b_m]}_{\text{input } \in\{0,1\}^m}) := h^w \text{ where } w = \prod_{i=1}^{m} x_i^{b_1}$$

State of the art algebraic PRFs

either

- Key-size quadratic in security parameter (NR-PRF).

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Nothing better for MACs known. Previous to this work no MAC construction from DDH with constant # of elements in key and constant # of exponentiations.

MACs vs. PRFs

MACs seem like simpler objects than PRFs

1. Unpredictability vs. indistinguishability.
2. Probabilistic vs. deterministic.

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1. Unpredictability vs. indistinguishability.
   Use search instead decision problems, CDH vs. DDH?
2. Probabilistic vs. deterministic.
   Easier from inherently probabilistic assumptions like LPN?

Definitions of MACs

$$\boxed{\text{TAG}(K,.)} \longleftarrow Q_T \longrightarrow \mathcal{A} \longleftarrow Q_V \longrightarrow \boxed{\text{VRFY}(K,.,.)}$$

uf-cmva : unforgeability under chosen message/verification attack

MAC $=$ {KG, TAG, VRFY} is (t, Q_T, Q_V, ϵ)-uf-cmva secure if for all adversaries $\mathcal{A}$ of size t making Q_T/Q_V TAG/VRFY queries: The probability $\mathcal{A}^{\text{TAG}(K,.),\text{VRFY}(K,.,.)}$ makes accepting VRFY query (M, ϕ) and TAG was not queried on M before is $\leq \epsilon$.

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Deterministic MAC with canonical verification.

1. $\text{TAG}(K, M)$ is deterministic.
2. $\text{VRFY}(K, M, \phi) = (\text{TAG}(K, M) \stackrel{?}{=} \phi)$

- No difference between 1 vs. many VRFY queries: $(t, Q_T, 1, \epsilon)$-uf-cmva $\Rightarrow (t, Q_T, Q_V, \epsilon Q_V)$-uf-cmva
- For probabilistic MACs 1 vs. many VRFY queries matters.

Selective security and Indistinguishability

uf-cma : is short for uf-cmva with one verification query

$$(t, Q_T, \epsilon)\text{-uf-cma} \stackrel{\text{def}}{=} (t, Q_T, 1, \epsilon)\text{-uf-cmva}$$

suf-cm(v)a : "selective" unforgeability, defined like uf-cm(v)a but where $\mathcal{A}$ must commit to forged message before making any oracle queries.

ind-cma : MAC is (t, Q_T, ϵ)-ind-cma if tags are indistinguishable

$$\left| \mathbb{P}_K[A^{\mathsf{TAG}(K,.)} = 1] - \mathbb{P}_K[A^{\mathsf{TAG}(K,0)}] \right| \leq \epsilon$$

Our Results (1) Transformations

Efficient generic transformation

1. From one to many verification queries
   uf-cma + ind-cma $\Rightarrow$ uf-cmva.

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1. From one to many verification queries
   uf-cma + ind-cma $\Rightarrow$ uf-cmva.
2. (trivial) Domain extension for uf-cma + ind-cma secure MACs.
3. (trivial) From selective to full security suf-cma $\Rightarrow$ uf-cma for MACs with small range.

Our Results (2) Constructions of algebraic MACs

General templates using

- CCA-secure pubilc-key encryption, Hash-proof systems.
- Key-homomorphic weak PRFs.
- Signatures schemes.

DL based Instantitations

construction	sk $\in$	Tag σ on m	Security	Assumption
$\mathsf{MAC}_{\mathsf{CS}}$	$\mathbb{Z}_p^4 \times \mathbb{G}$	$\mathbb{G}^4$	uf-cmva	DDH
$\mathsf{MAC}_{\mathsf{HPS}}$	$\mathbb{Z}_p^3$	$\mathbb{G}^3$	uf-cmva	DDH
$\mathsf{MAC}_{\mathsf{hwPRF}}$	$\mathbb{Z}_p^2$	$\mathbb{G}^2$	suf-cma	DDH
$\mathsf{MAC}_{\mathsf{WhwPRF}}$	$\mathbb{Z}_p^{\lambda+2}$	$\mathbb{G}^2$	uf-cma	DDH
$\mathsf{MAC}_{\mathsf{BB}}$	$\mathbb{Z}_p^3$	$\mathbb{G}^2$	suf-cma	gap-CDH
$\mathsf{MAC}_{\mathsf{TBB}}$	$\mathbb{Z}_p^5$	$\mathbb{G}^3$	suf-cma	CDH
$\mathsf{MAC}_{\mathsf{Waters}}$	$\mathbb{Z}_p^{\lambda+2}$	$\mathbb{G}^2$	uf-cmva	gap-CDH
$\mathsf{PRF}_{\mathsf{NR}}$	$\mathbb{Z}_p^{\lambda} \times \mathbb{G}$	$\mathbb{G}$	PRF	DDH

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From [KPCJV11]

construction	sk $\in$	Tag σ on m	Security	Assumption
$\mathsf{MAC}_{\mathsf{LPN}}$	$\mathbb{Z}_2^{2\ell}$	$\mathbb{Z}_2^{(\ell+1)\times n}$	suf-cma	LPN
$\mathsf{MAC}_{\mathsf{BilinLPN}}$	$\mathbb{Z}_2^{\ell\times\lambda}$	$\mathbb{Z}_2^{(\ell+1)\times n}$	uf-cma	LPN

Transformations

From one to many verification queries

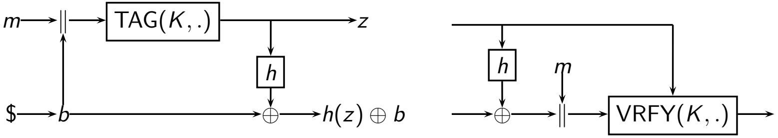

Figure: $\overline{\text{TAG}}$ and $\overline{\text{VRFY}}$ with key (K, h) for message m using randomness b. h is pairwise independent with range $\{0, 1\}^{\mu}$.

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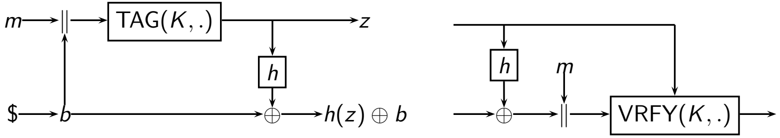

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Theorem (uf-cma + ind-cma ⇒ uf-cmva)

For any $t, Q_T, Q_V \in \mathbb{N}$, $\epsilon > 0$, *if* MAC *is*

- (t, Q_T, ϵ)-uf-cma *secure*
- (t, Q_T, ϵ)-ind-cma *secure*

then $\overline{\text{MAC}}$ *is* (t, Q_T, Q_V, ϵ')-uf-cmva *secure where*

$$\epsilon' = 2Q_V\epsilon + 2Q_V Q_T/2^{\mu}.$$

From selective to full security & domain extension

Selective to full security

Any MAC with message domain $\{0,1\}^\mu$

$$(t, Q, \varepsilon)\text{-suf-cma} \quad \Rightarrow \quad (t, Q, \varepsilon 2^\mu)\text{-uf-cma}$$

Domain Extension

Pairwise independent $g : \{0,1\}^m \to \{0,1\}^\mu$ to increase domain.

$$\mathsf{TAG}'(K, M) = \mathsf{TAG}(K, g(M))$$

$$\begin{array}{rcl} (t, Q, \varepsilon)\text{-uf-cma} & \& & (t, Q, \varepsilon)\text{-ind-cma} \\ & \Rightarrow & \\ (t, Q, 2\varepsilon + Q/2^\mu)\text{-uf-cma} & \& & (t, Q, \varepsilon)\text{-ind-cma} \end{array}$$

Constructions

Construction from key-homomorphic weak PRF

Key-homomorphic weak PRF

Keyed family of functions $\{f_k \ : \ \mathcal{X} \to \mathcal{Y}\}_{k \in \mathcal{K}}$.

1. wPRF: $f_k(.)$ indistinguishable from random *on random inputs.*
2. key-homomorphic: $f_{a \cdot k_1 + b \cdot k_2}(x) = a \cdot f_{k_1}(x) + b \cdot f_{k_2}(x)$.

kwPRF from DDH

$\{f_k \ : \ \mathbb{G} \to \mathbb{G}\}_{k \in \mathbb{Z}_p}$ defined as $f_k(x) = x^k$.

1. wPRF under DDH.
2. key-homomorphic:
   $f_{a \cdot k_1 + b \cdot k_2}(x) = x^{a \cdot k_1 + b \cdot k_2} = (f_{k_1}(x))^a (f_{k_2}(x))^b$.

Construction from key-homomorphic weak PRF

$\{f_k \;:\; \mathcal{X} \mapsto \mathcal{Y}\}_{k\in\mathcal{K}}$

$$\begin{aligned} \mathsf{KG}:&\quad k_1, k_2 \in_\$ \mathcal{K}. \\ \mathsf{TAG}_{(k_1,k_2)}(m):&\quad x, f_{m\cdot k_1+k_2}(x)\;,\; x \in_\$ \mathcal{X} \\ \mathsf{VRFY}_{(k_1,k_2)}(m,(x,y)):&\quad f_{m\cdot k_1+k_2}(x) \stackrel{?}{=} y. \end{aligned}$$

Theorem

If f is a key-homomorphic weak PRF then MAC *is* suf-cma *and* ind-cma *secure MAC.*

Instantiation with DDH

$$\begin{aligned} \mathsf{KG}:&\quad k_1, k_2 \in_\$ \mathbb{Z}_p \\ \mathsf{TAG}_{(k_1,k_2)}(m):&\quad x, x^{m\cdot k_1+k_2}\;,\; x \in_\$ \mathbb{G} \\ \mathsf{VRFY}_{(k_1,k_2)}(m,(x,y)):&\quad x^{m\cdot k_1+k_2} \stackrel{?}{=} y \end{aligned}$$

Constructions from signatures

- uf-cma secure signature scheme is a uf-cmva secure MAC.

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$\mathsf{MAC}_{\mathsf{BB}}$ from downgraded BB (prime-order instead bilinear group)

$$\begin{aligned}
\mathsf{KG}: &\ k = (x, x', y) \in_{\$} \mathbb{Z}_p^3. \\
\mathsf{TAG}_k(m): &\ (U, g^{xy} \cdot U^{xm+x'}) \in \mathbb{G}^2 \text{ where } U \in_{\$} \mathbb{G}. \\
\mathsf{VRFY}_k(m, (U, V)): &\ g^{xy} \cdot U^{xm+x'} \stackrel{?}{=} V.
\end{aligned}$$

Theorem

If gap-CDH holds in $\mathbb{G}$ then $\mathsf{MAC}_{\mathsf{BB}}$ is suf-cma secure.

Constructions from signatures

- uf-cma secure signature scheme is a uf-cmva secure MAC.
- Overkill as MACs don't need public verification.
- Take signature scheme and "downgrade" it: loose public verifiability but gain efficiency.
- Can go from gap-CDH to CDH using twinning Cash et. al EC'08.

$\mathsf{MAC}_{\mathsf{TBB}}$ downgraded BB plus twinning

$$\mathsf{KG}: \quad k = (x_1, x_1', x_2, x_2', y) \in_\$ \mathbb{Z}_p^5.$$

$$\mathsf{TAG}_k(m): \quad U, g^{x_1 y} U^{x_1 m + x_1'}, g^{x_2 y} U^{x_2 m + x_2'} \text{ where } U \in_\$ \mathbb{G}.$$

$$\mathsf{VRFY}_k(m, (U, V)): \quad g^{xy} \cdot U^{xm + x'} \stackrel{?}{=} V.$$

Theorem

If CDH holds in $\mathbb{G}$ then $\mathsf{MAC}_{\mathsf{TBB}}$ *is* suf-cma *secure.*

Construction from LPN [KPCJV11]

MAC_{TBB} downgraded BB plus twinning

$$\text{KG}: \ \mathbf{x} \in_\$ \mathbb{Z}_2^{2\ell}$$

$$\text{TAG}_k(\mathbf{m}): \ (\mathbf{R}, \mathbf{R}^T \cdot \mathbf{x}_{\downarrow \mathbf{m}} + \mathbf{e}) \text{ where } \mathbf{R} \in_\$ \mathbb{Z}_2^{\ell \times n} \text{ and } \mathbf{e} \in \mathbb{Z}_2^n \text{ has low weight.}$$

$$\text{VRFY}_k(\mathbf{m}, (\mathbf{R}, \mathbf{z})): \ |\mathbf{R}^T \cdot \mathbf{x}_{\downarrow \mathbf{m}} - \mathbf{z}| \text{ has low weight.}$$

Theorem (KPCJV11)

If LPN is hard, then MAC_{LPN} *is* suf-cma *and* ind-cma.

Questions?